Information Processing in Neuron with Exponential Distributed Delay

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**Abstract**

Artificial intelligence (AI) has become the primary need in nearly all sectors namely engineering, services, banking, finance, defense, space etc [3], [33]. Artificial intelligence in these sectors can be implemented in two ways: (i) hardware level implementation (ii) software level implementation. Both kinds of AI implementation require neuron models which mimic the minimal set of real neuron functionality. To this end, Leaky Integrate-and-Fire (LIF) model is performing as the backbone for both kinds of AI implementation.

At hardware level implementation, it’s a variant, called as neuristors, is used at chip level implementation, whereas a number of variants LIF model are used to implement AI at software level. In this work, the extended LIF model in distributed delay kernel regime is analyzed. The impact of exponentially distributed delay (EDD) memory kernel on spiking activity and steady state membrane potential distribution (SVD) of LIF neuron is investigated. Fokker-Planck equation associated with the considered model is solved to investigate SVD of the neuron in sub-threshold regime, which results Gaussian distribution. In order to study the information processing, spiking activity of the model is investigated, which is further extended to neuronal rate-code scheme. These finding have been compared with simple LIF model with stochastic input. It is evident that steady state membrane potential distribution of the LIF neuron is invariant due to the presence of EDD. Such kinds of neuron models are useful to implement artificial neural networks. To this end, the proposed model can used to implement recurrent neural networks (RNN) with comparatively more accuracy. Similarly, this model can also be investigated in term of chip level implementation of AI.

**Keywords**

Artificial Intelligence, Distributed Delay, Fokker-Planck Equation, LIF Model, LSTM, Spiking Activity, Recurrent Neural Network, Steady State Probability Distribution

**1. Introduction**

Artificial intelligence is computer based system comprised of neural networks, which behaves like human brain [29]. Neural networks are artificial human-brain like computing structure which incorporates

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the membrane information processing mechanism [7], [9]. Long short-term memory (LSTM) network, recurrent neural network are two important neural networks which deals with the memory element. Neuron forms basis for information processing in these networks and generates the real central nervous system information processing behavior [1] [7]. A biological neuron has three fundamental information processing entities, namely, soma, axons and dendrites [7] [28]. Dendrite and axon meet at synapse to establish a communication link between two neurons [1], [28]. Neuron generates action potential in form spike sequences which travel through axon down the nervous system [4], [15]. In this way, a neuron forms a computational device which maps synaptic input into spikes sequence. This has motivated to researchers to study the correspondence among input stimuli and spiking patterns. Computational neuroscience provides a platform to understand brain information processing functions in terms of representation, information processing as chemical and electrical signals, effect of memory element [26] so that artificial implementation become more robust and accurate. Information theoretic framework includes mechanisms to measure uncertainty between encoding and decoding of spiking patterns in quantitative means [4], [7], [13].

Lapicque (1907) has proposed the integrate-and-fire neuron model (IF model), first neuron model, equivalent to an electrical RC-circuit with an additional threshold constraint, i.e. neuron potential resets a lower value as soon as it reaches to a certain value (threshold) [1], [7], [9]. Under the influence of noisy input stimulus and noisy environment, neuron reflects the highest level of variation in spiking pattern [25]. Sub-threshold and super-threshold regime are two set of parameters in neuronal information processing which covers the noise. Neuron remains silent in former regime whereas it emits spikes in the late regime under no influence of noise [17].

Memory elements can be introduced in a neuron model via kernel function. Karmeshu et. al. [20] has proposed a distributed delay framework and has made an enlarged study of LIF model for exponential distributed delay kernel function and noticed oscillatory spiking patterns in inter-spike-interval distribution [20]. In this article, LIF model with exponentially distributed kernel function is further investigated in terms of spiking activity and SVD. Simulation based study is done to study the spiking patterns and SVD is computed by solving the associated Fokker-Planck equation (FPE). These findings are compared with stochastic LIF model.

The article is organized into six sections. After a brief introduction of neural network, neuron model and its information processing mechanism in Section I, neuronal DDF is explained in Section II. Section III deals with computation of SVD. Spiking activity is analyzed via neuronal information processing rate-code scheme in Section IV. Section V discusses about artificial intelligence in terms of various used techniques and its application. Finally, Section VI contains analysis of findings and conclusion.

2. Distributed Delay Framework Of Neuron Model

Membrane potential is the key parameter for neuron models. Variation in membrane potential is always taken care to deal with neuron’s behavior. LIF model is an extended version of IF model. It assume that the membrane conductance is leaky i.e. if there is no electrical activity happening to and fro to the neuron, then membrane potential will get decayed with respect to time. To and for activity of membrane potential in a neuron occur due to the movement of various ions and molecules. The aggregated effect of these ions and molecules can be captured in term of stochastic current. Thus, the rate of change of membrane potential ($V(t)$) for LIF model with stochastic input stimulus is defined as [5], [6], [7], [10].

$$\frac{dV(t)}{dt} = -\beta V(t) + \mu + \xi(t)$$  \hspace{1cm} (1)

Here, $\beta$ is membrane decay constant, $\mu$ is mean input stimulus, $\xi(t)$ is delta correlated Gaussian White noise with intensity , i.e. $\langle \xi(t) \rangle = 0$ and $\langle \xi(t_i)\xi(t_j) \rangle = \frac{\sigma^2}{2}$. DDF for the LIF model is defined as [7], [20]

$$\frac{dV(t)}{dt} = -\beta \int_0^t K(t-\tau)V(\tau)d\tau + \mu + \xi(t)$$  \hspace{1cm} (2)
Here $K(t)$ is distributed delay kernel. This captures the effect of past membrane potentials on the current values. $K(t)$ can has multiple forms, like Gamma distribution, Sigmoid, Laplace distributed etc. Substitution of gamma distributed delay kernel for $K(t)$ in Eq. (2) results as below [20].

$$
\frac{dV(t)}{dt} = -\beta \int_0^t \frac{\Gamma^{m+1}(t-\tau)^m e^{-\eta(t-\tau)}}{m!} V(\tau) d\tau + \mu + \xi(t)
$$

(3)

Here $\eta$ and $m$ are gamma distributed kernel parameters. Eq. (3) reduces into LIF model with exponential distributed delay kernel for $m=0$. Exponential distributed delay kernel is also known as the weak-kernels, as it captures only the first order of membrane potential delay dependency. This results membrane potential evolution process $\{V(t); t \geq 0\}$ into a non-Markovian process. Analytical analysis of a non-Markovian process is a too complex and has to look on some other methods. The present non-Markovian process can be extended into higher dimensions to transform membrane potential evolution into Markovian process [20]. LIF model with EDD in extended space takes the following form [20].

$$
\frac{dV}{dt} = -\{\eta \beta U_0(t) - \mu\} + \xi(t)
$$

(4.1)

$$
\frac{dU_0}{dt} = -\eta\{U_0(t) - V(t)\}
$$

(4.2)

with initial conditions $V(t)=V_0, U_0(t)=0$ at $t=0$.

Eqs. (4.1) and (4.2) form system of coupled stochastic differential equations and can analyzed analytically. Ito method and Stratanovich method are two important techniques to numerically study stochastic differential equations. The primary difference in both techniques is the point of approximation in the sub-time interval. Ito technique uses left bound of sub-time interval whereas Strat anovich techniques uses mean of the left and right bounds of sub-time interval [14].

3. SVD for LIF Neuron with EDD Kernel Function

Rate of change of spatial probability of a stochastic variable are studied in terms of Fokker-Planck equations (FPEs). For the system of coupled stochastic Eqs. (4.1) and (4.2), FPE results membrane potential probability distribution [5], [6], [31]. FPE is also used to solve first passage time problem, firing rate computation and estimation of moments of stochastic conducance’s [21], [23]. Solution of Fokker-Planck equation highly depends on the boundary conditions [14]. FPE solution highly depends on initial conditions and boundary conditions. When membrane potential reaches at threshold value, the probability of generation of spike is 1 for absorbing boundary condition, whereas, it is 0 for reflecting boundaries. In later condition, probability current flux $J(V, t)$ becomes equal to zero. The extended LIF model, defined in Eqs. (4.1) and (4.2) are analyzed with reflecting boundary condition.

Let $p^E(V, U_0, t)$ is membrane potential probability distribution in extended apace. Following Burkitt [5], [6] and Frank [14], corresponding Fokker-Planck equation becomes

$$
\frac{dp^E}{dt} = \frac{\partial}{\partial V} (\beta U_0 - \mu)p^E + \frac{\partial}{\partial U_0} \eta(U_0 - V(t))p^E + \frac{\sigma^2}{2} \frac{\partial^2 p^E}{\partial V^2}
$$

(5)

With boundary conditions: $p^E(V, U_0, t | t = 0) = \delta(t-t_0)\delta(U_0(t)-U_0(t_0))\delta(V(t)-V(t_0))$

and $p^E(V, U_0, 0) = Vp^E(V, U_0, t | t = 0) = 0$ as $V \rightarrow \infty$. 

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Eq. (5) can be rewritten with differential operator \( \nabla = (\frac{\partial}{\partial V}, \frac{\partial}{\partial U_0}) \) as

\[
\frac{\partial p^E}{\partial t} = \nabla (Ap^E + (\nabla (Bp^E))^T) ; \text{ with } A = \begin{pmatrix} \beta U_0 - \mu \\ \eta (U_0 - V) \end{pmatrix} \text{ and } B = \begin{pmatrix} \frac{\sigma^2}{2}, 0 \\ 0, 0 \end{pmatrix}
\]

(6)

The probability current flux takes the form

\[
J = \nabla (Ap^E + (\nabla (Bp^E))^T)
\]

(7)

In case of SVD, \( \frac{\partial p^E}{\partial t} = 0 \), \( p^E = p_s^E \) and for reflecting boundaries, Eq. (7) yields

\[
Ap^E + (\nabla (Bp^E))^T = 0
\]

(8)

Substituting of \( A, B \) and using matrix calculation in Eq. (8) yields

\[
\begin{pmatrix} \beta U_0 - \mu \end{pmatrix} p_s^E + \frac{\partial}{\partial V} \left( \frac{\sigma^2 p_s^E}{2} \right) + \eta (U_0 - V) = 0
\]

(9)

Its simplification further results

\[
\frac{1}{p_s^E} \frac{\partial p_s^E}{\partial V} = -\frac{\sigma^2}{2} (\beta V - \mu)
\]

(10)

Integration of Eq. (10) results SVD as

\[
p_s^E = K_E \exp\left\{ -\frac{\beta}{\sigma^2} \{ (V - \mu)^2 - \mu^2 \} \right\}
\]

(11)

Law of conservation of probabilities can be used to compute the normalization constant \( K_E \) [12]. Eq. (11) is the stationary state membrane potential distribution of the LIF model in exponentially distributed delay kernel. This membrane potential distribution has the Gaussian distributed form. Similarly, SVD for LIF neuron with stochastic input stimulus is computed below.

Let \( p^L(V, t) \) is the spatial probability distribution of membrane potential for LIF model. FPE associated with Eq. (1) is given as

\[
\frac{dp^L}{dt} = \frac{\partial}{\partial V} (\beta V - \mu) p^L + \frac{\sigma^2}{2} \frac{\partial^2 p^L}{\partial V^2}
\]

(12)

With boundary conditions:

\[
p^L(V, t \mid t = 0) = \delta(t-t_0)\delta(V(t)-V(t_0)) \text{ and } p^L(V, 0) = V p^L(V, t \mid t = 0) = 0 \text{ as } V \rightarrow \infty.
\]

Probability current flux \( J \) for Eq. (12) becomes

\[
J = (\beta V - \mu) p^L + \frac{\sigma^2}{2} \frac{\partial p^L}{\partial V}
\]

(13)
For SVD of neuron, \( \frac{\partial p^L_s}{\partial t} = 0 \), \( p^L = p^L_s \). Simplification of Eq. (13) for reflecting boundaries results

\[
\frac{1}{p^L_s} \frac{\partial p^L_s}{\partial V} = -\frac{\sigma^2}{2} (\beta V - \mu)
\]  
(14)

Integration of Eq. (14) yields SVD with normalization constant \( K_L \) as

\[
p^L_s = K_L \exp\left\{-\frac{\beta}{\sigma^2} \left\{ (V - \mu)^2 - \mu^2 \right\}\right\}
\]  
(15)

Eq. (15) is the stationary state membrane potential distribution of the LIF model with stochastic input stimulus. This membrane potential distribution is also Gaussian distributed and similar with the SVD of LIF model with exponential distributed delay kernel.

4. Information Processing in a Neuron with Exponential Distributed Delay

Neuron encodes information in term of spikes. Rate code scheme and temporal code scheme are two neuron information encoding scheme [2], [13], [17], [18], [19], [27], [28]. Rate code scheme uses average number of spikes fired by neuron to encode information [15], [16], [17]. Time interval between two consecutive spikes (inter-spike-interval) is used to encode information in temporal coding scheme [17], [25].

Information processing of LIF neuron in EDD is analyzed with rate code scheme. Euler-Maruyama numerical simulation technique for stochastic differential equation [19] is used to simulate neuronal models represented in Eq. (1) and Eq. (4). The average number of spikes per sec fired by neuron (Spike Count) with different values of parameters \( \mu, \sigma \) and \( \eta \) is calculated. In each iteration of simulation, averaging is done for 1000 trials of simulation with 10000 msec time in each trial. Findings are illustrated in Fig. 1-Fig. 4.

Parameter value \( \beta \) is taken ranging from 0.01 to 0.3 with step-size of 0.01 [7], [8]. Spike rate for LIF model in EDD is similar to that of LIF model with stochastic input at small \( \beta \) and reduces in LIF model in EDD is slowly than LIF model with stochastic input for increment in \( \beta \) as shown in Fig. 1. Lowering in applied input stimulus \( \mu \) by half, firing rate for both models reduces, but, the firing activity of LIF model stops for larger values of \( \beta \) whereas, LIF model with EDD remains spiking. This observation is shown depicted in Fig. 2. Similar spiking activity has been observed neuronal disorder like schizophrenia [32].

![Fig. 1. Spike Count for \( \mu = 0.2 \), \( \sigma = 0.05 \) and \( \eta = 0.1 \)]
Spiking activity of LIF model with distributed delay decreases with the increase in delay parameter $\eta$ as shown in Fig. 3 and Fig. 4. For $\eta = 1$, spiking activity of both models result similar patterns. In Fig. 4, it is well illustrated that spiking activity of simple LIF model and LIF model with exponential distributed delay approximately equal which provides the critical values for parameters $\eta$ at which effect of delay kernel function on evolution of membrane potential becomes negligible.

5. Discussion

Artificial intelligence is a way to transform machines as intelligent machines. This can be done at hardware as well as software level. AI has improved human life in terms of manufacturing automation and service performances in past 20 years [29]. Intelligent machines and systems are also known as expert systems. Expert systems are widely used in business, engineering, medicine, science, security, space,
weather forecasting etc. to solve complex problems [29], [33], [11]. McCulloch and Pitts have used artificial neurons to study the AI with logical modeling, first time, in 1965 [29]. Since then, multiple techniques have been evolved to incorporate AI in systems and machines. Few important AI techniques are expert systems, fuzzy logic, artificial neural networks (ANN), multi-layered detectors and learning algorithm and genetic algorithm [29]. Expert system uses person’s area of expertise and optimizes the use of their skills. It is highly applicable in biomedicine, logistics, marketing, medicines and retail fields. Fuzzy logic and corresponding neural networks uses back propagation and superiorities in combination to incorporate learning and decision-making. It is widely used in time-sequence calculations. Artificial neural network mimics real neuron comparatively more similar way. First of all these networks learns from past data which is termed as training, which includes network structures with different types of connection among multiple artificial neurons. During training process, these connections obtain a weight (synaptic strength) which is used in later stage decision makings. Multiple-layered detectors and learning algorithms involves ANN with more than two artificial neuron layers. Such kind of networks are also termed as deep learning networks and are being used to obtain decision making in multi-class classification problems. Genetic algorithm uses natural selection mechanism and is a probability based AI technique. Genetic algorithms are used to complex problem in economic, information system, mechanical learning, social systems etc. Few important area of AI application is given as below.

- Computer Games: AI helps to artificial character in finding his movement path, making a decision for action and learning from previous movements.
- Banking and Finance Sector: AI helps to find fraud-detection and generating multiple recommendations.
- Medicine: AI helps to clinical support system, medical image classification, diagnostic analysis, MRI brain tumor detection etc.
- Security: AI helps in security in many folds. It is easier to monitor and provide security for human and places with the combination of AI and computer vision. AI is also useful in network monitoring and security in term of intrusion detection and prevention.

6. Conclusion and Future Scope

SVD of LIF neuron in EDD (Eq.(11)) is Gaussian distribution and is asymptotically similar to SVD of LIF neuron with stochastic input (Eq.(15)) i.e. limit \( V \to \infty, p_s^E = p_s^L \). This finding suggests that the exponentially distributed delay kernel function has no effect on stationary state membrane potential of LIF neuron in it’s sub-threshold regime. LIF model with stochastic input stimulus and it’s extended form in distributed delay regime, both, represents similar spiking activity. Simulation based analysis results that the spike count of neuron increases for EDD. EDD behaves as a memory element and helps neuron to reach at threshold value in quicker time. Masas [24] has suggested a mechanism for self organization and learning from noise in neural networks. EDD has virtue to include memory element which can further be extended to model neural networks. Lim et. al. [22] has suggested a mechanism for hardware level implementation of artificial neural network (ANN) which can further be experimented in EDD.

References


Author’s Biography

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